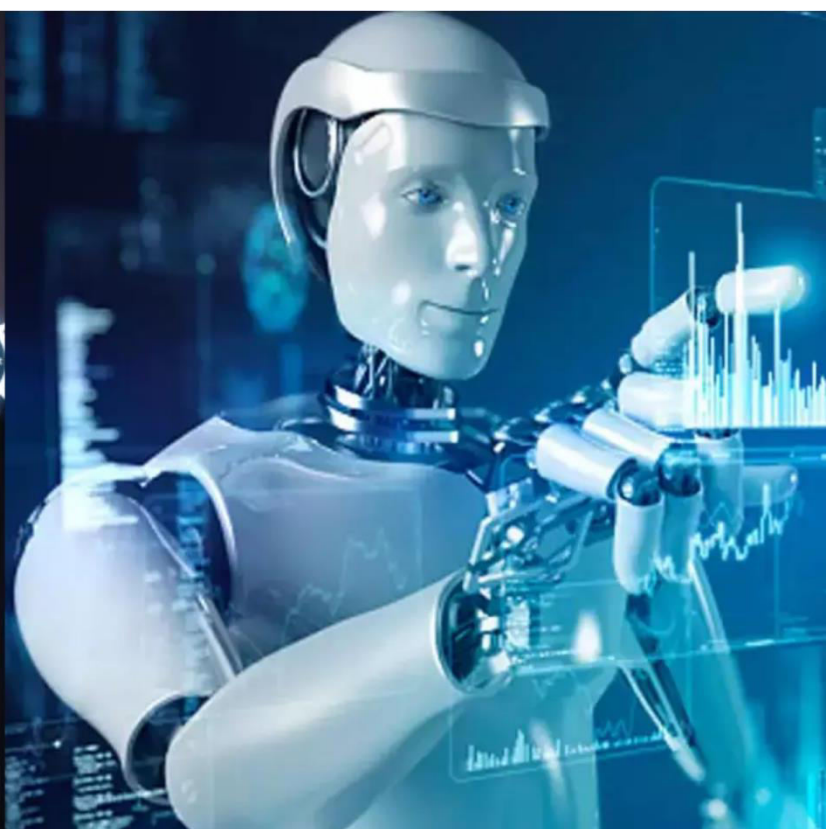




International Journal of Innovative Research in Science Engineering and Technology (IJIRSET)

(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)



Impact Factor: 9.935

Volume 15, Issue 9, September 2026

Lattice Valued Picture Fuzzy Sets

Tweena Evangelin P A., Francina Shalini²

PG and Research Department of Mathematics, Nirmala College for Women, Coimbatore, Tamil Nadu, India^{1,2}

ABSTRACT: In this paper, the definition of Lattice Valued Picture Fuzzy Set (LVPFS) has been introduced by inducing the concept of lattice to picture fuzzy set. Some of the basic set operations for the Lattice Valued Picture Fuzzy sets are defined and their properties are investigated with examples.

KEYWORDS: Lattice, Picture Fuzzy Set, Set Operations, Lattice valued Picture Fuzzy Set.

I. INTRODUCTION

Most real-world issues involve ambiguity and uncertainty in the data. In many disciplines, including Engineering, medical sciences and Economics, handling such data is a major problem. To address this concern, Zadeh [1] introduced fuzzy sets. Cases when an element partially fits into a set were handled well by fuzzy sets. However, inadequate data led to uncertainties in certain circumstances that fuzzy sets were unable to address. Atanassov [2] proposed idea of Intuitionistic fuzzy sets. Neutrosophic Sets had been introduced by Smarandache [3]. Following this, Cuong [4] described a novel concept by adding a grade of neutral membership and termed it Picture Fuzzy Set. The Inconsistent Intuitionistic Fuzzy Sets (Ternary Fuzzy Set and Picture Fuzzy Set) are special cases of Neutrosophic Set [5]. Yuan and Wu [6] induced fuzzy set's concept in lattice theory. The concept of Lattice Valued Intuitionistic Fuzzy Set was introduced by T. Gerstenkorn, A. Tepavčević [7]. Jose James and Sunil C. Mathew [8] introduced Lattice valued Neutrosophic Sets.

In the current article, we define LVPFS and extend notion of picture fuzzy sets to the theory of lattices. The definition of set operations and an analysis of their associated properties have been presented.

II. PRELIMINARIES

Definition 2.1: [4] A picture fuzzy set $A = \{(\hat{a}, \mu_A(\hat{a}), \eta_A(\hat{a}), \nu_A(\hat{a})) | \hat{a} \in \hat{X}\}$ where $\mu_A(\hat{a}), \eta_A(\hat{a}), \nu_A(\hat{a}) \in [0,1]$ are called the positive, neutral and negative membership degrees of $\hat{a}$ in $\hat{X}$ and satisfy the following condition: $\forall \hat{a} \in \hat{X}, \mu_A(\hat{a}) + \eta_A(\hat{a}) + \nu_A(\hat{a}) \leq 1$. Now $1 - (\mu_A(\hat{a}) + \eta_A(\hat{a}) + \nu_A(\hat{a}))$ could be called the degree of refusal membership of $\hat{a} \in \hat{X}$.

Definition 2.2: [4] The union, intersection, and complement for each pair of PFSs A and B are described as follows:

1. $A \subseteq B$ iff $\forall x \in X, \mu_A(x) \leq \mu_B(x), \eta_A(x) \leq \eta_B(x), \nu_A(x) \geq \nu_B(x)$ (1)
2. $A = B$ iff $A \subseteq B$ and $B \subseteq A$ (2)
3. $A \cup B = \{ \langle x, \max(\mu_A(x), \mu_B(x)), \min(\eta_A(x), \eta_B(x)), \min(\nu_A(x), \nu_B(x)) \rangle | x \in X \}$ (3)
4. $A \cap B = \{ \langle x, \min(\mu_A(x), \mu_B(x)), \min(\eta_A(x), \eta_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle | x \in X \}$ (4)
5. $\bar{A} = \{ \langle x, \nu_A(x), \eta_A(x), \mu_A(x) \rangle | x \in X \}$ (5)

Definition 2.3: [9] Let P be a non-empty ordered set. If $x \vee y$ and $x \wedge y$ exist for all $x, y \in P$, then P is called a lattice.

Definition 2.4: [10] It "is said that a lattice L is distributive" if $\forall x, y, z \in L, x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ and

$$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z).$$

Definition 2.5: [9] Consider L "be a complete lattice & $A, B \subseteq L$."

1. $\forall a \in L$ and $\forall B \subseteq L, a \wedge (\bigvee B) = \bigvee_{b \in B} (a \wedge b)$ (6)
2. $\forall a \in L$ and $\forall B \subseteq L, a \vee (\bigwedge B) = \bigwedge_{b \in B} (a \vee b)$ (7)

Definition 2.6: [9] For a lattice $(L, \leq)$, for any $\forall a, b \in L$,

$$1. \quad a \vee b = b \Leftrightarrow a \leq b \quad (8)$$

$$2. \quad a \wedge b = a \Leftrightarrow a \leq b \quad (9)$$

Definition 2.7: [9] Let $(L, \vee, \wedge, 0, 1)$ be a bounded lattice with 0 & 1 denoting minimum as well as maximum elements correspondingly & let $a \in L$. Then $b \in L$ is known as a complement of a if $a \vee b = 1$ and $a \wedge b = 0$. A bounded lattice with complement for each member is called complemented lattice.

Definition 2.8: [9] If $(L, \vee, \wedge)$ is a complemented lattice in which each element has a unique complement, then unique complement of a is represented by a' .

Definition 2.9: [9] We refer to a complemented distributive lattice as a Boolean lattice.

III. LATTICE VALUED PICTURE FUZZY SET

Definition 3.1: Let $\wp$ be the universe and $\mathcal{L}$ be a nontrivial complete boolean lattice. A Lattice Valued Picture Fuzzy Set (LVPFS) $\mathfrak{C}_{\mathcal{L}} = \{ \langle \dot{u}, \zeta_{(\mathfrak{C}_{\mathcal{L}})}(\dot{u}), \iota_{(\mathfrak{C}_{\mathcal{L}})}(\dot{u}), \vartheta_{(\mathfrak{C}_{\mathcal{L}})}(\dot{u}) \rangle \mid \dot{u} \in \wp \}$ where $\zeta_{(\mathfrak{C}_{\mathcal{L}})}: \wp \rightarrow \mathcal{L}$ is positive membership degree, $\iota_{(\mathfrak{C}_{\mathcal{L}})}: \wp \rightarrow \mathcal{L}$ is the neutral membership degree and $\vartheta_{(\mathfrak{C}_{\mathcal{L}})}: \wp \rightarrow \mathcal{L}$ is the negative membership degree. $\pi_{(\mathfrak{C}_{\mathcal{L}})}(\dot{u}) = (\zeta_{(\mathfrak{C}_{\mathcal{L}})}(\dot{u}) \vee \iota_{(\mathfrak{C}_{\mathcal{L}})}(\dot{u}) \vee \vartheta_{(\mathfrak{C}_{\mathcal{L}})}(\dot{u}))'$ is the refusal membership degree of $\dot{u}$ in $\mathfrak{C}_{\mathcal{L}}$.

Remark 3.2: All lattices we consider here are assumed to contain the lower bound $0_{\mathcal{L}}$ and the upper bound $1_{\mathcal{L}}$.

Example 3.3: Let $\wp = \{\dot{u}_1, \dot{u}_2\}$ and the lattice as in Fig.1.

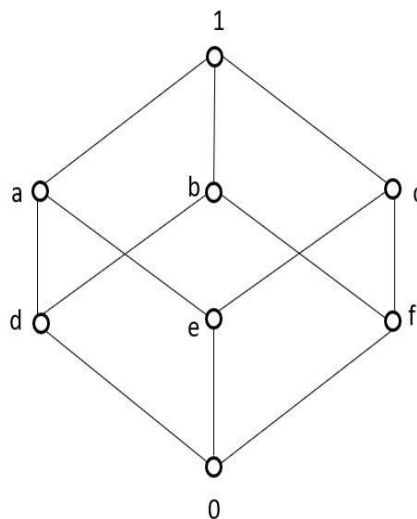


FIGURE.1. Representation of Boolean lattice of order 3

$\mathfrak{C}_{1_{\mathcal{L}}} = \{ \langle \dot{u}_1, c, e, 1 \rangle, \langle \dot{u}_2, c, f, e \rangle \}$ and $\mathfrak{C}_{2_{\mathcal{L}}} = \{ \langle \dot{u}_1, 1, a, a \rangle, \langle \dot{u}_2, c, b, 0 \rangle \}$ are LVPF sets in $\wp$.

Definition 3.4: Let $\mathfrak{C}_{1_{\mathcal{L}}}, \mathfrak{C}_{2_{\mathcal{L}}} \in \text{LVPFS}(\wp)$. Then $\mathfrak{C}_{1_{\mathcal{L}}} \subseteq \mathfrak{C}_{2_{\mathcal{L}}}$ iff $\forall \dot{u} \in \wp, \zeta_{(\mathfrak{C}_{1_{\mathcal{L}}})}(\dot{u}) \leq \zeta_{(\mathfrak{C}_{2_{\mathcal{L}}})}(\dot{u}), \iota_{(\mathfrak{C}_{1_{\mathcal{L}}})}(\dot{u}) \leq \iota_{(\mathfrak{C}_{2_{\mathcal{L}}})}(\dot{u}), \vartheta_{(\mathfrak{C}_{1_{\mathcal{L}}})}(\dot{u}) \geq \vartheta_{(\mathfrak{C}_{2_{\mathcal{L}}})}(\dot{u})$. $\mathfrak{C}_{1_{\mathcal{L}}} = \mathfrak{C}_{2_{\mathcal{L}}}$ iff $\mathfrak{C}_{1_{\mathcal{L}}} \subseteq \mathfrak{C}_{2_{\mathcal{L}}}$ and $\mathfrak{C}_{2_{\mathcal{L}}} \subseteq \mathfrak{C}_{1_{\mathcal{L}}}$.

Definition 3.5: The complement of a LVPF set $\mathfrak{C}_{\mathcal{L}}$ is $\mathfrak{C}_{\mathcal{L}}^c = \{ \langle \dot{u}, \vartheta_{(\mathfrak{C}_{\mathcal{L}})}(\dot{u}), \iota_{(\mathfrak{C}_{\mathcal{L}})}(\dot{u}), \zeta_{(\mathfrak{C}_{\mathcal{L}})}(\dot{u}) \rangle \mid \dot{u} \in \wp \}$.

Definition 3.6: Let $\mathfrak{C}_{1L}, \mathfrak{C}_{2L} \in \text{LVPFS}(\mathcal{P})$. Then $\forall \dot{u} \in \mathcal{P}$,

$$\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L} = \{ \langle \dot{u}, \zeta_{(\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L})}(\dot{u}), \iota_{(\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L})}(\dot{u}), \vartheta_{(\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L})}(\dot{u}) \rangle \mid \dot{u} \in \mathcal{P} \}$$

where

$$\zeta_{(\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L})}(\dot{u}) = \zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \vee \zeta_{(\mathfrak{C}_{2L})}(\dot{u}), \quad (10)$$

$$\iota_{(\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L})}(\dot{u}) = \iota_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \iota_{(\mathfrak{C}_{2L})}(\dot{u}), \quad (11)$$

$$\vartheta_{(\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L})}(\dot{u}) = \vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \vartheta_{(\mathfrak{C}_{2L})}(\dot{u}). \quad (12)$$

$$\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L} = \{ \langle \dot{u}, \zeta_{(\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L})}(\dot{u}), \iota_{(\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L})}(\dot{u}), \vartheta_{(\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L})}(\dot{u}) \rangle \mid \dot{u} \in \mathcal{P} \} \text{ where}$$

$$\zeta_{(\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L})}(\dot{u}) = \zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \zeta_{(\mathfrak{C}_{2L})}(\dot{u}), \quad (13)$$

$$\iota_{(\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L})}(\dot{u}) = \iota_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \iota_{(\mathfrak{C}_{2L})}(\dot{u}), \quad (14)$$

$$\vartheta_{(\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L})}(\dot{u}) = \vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) \vee \vartheta_{(\mathfrak{C}_{2L})}(\dot{u}). \quad (15)$$

Here, $\vee$ and $\wedge$ denote the join (or supremum) and meet (or infimum) of the lattice respectively.

Theorem 3.7: Let $\mathfrak{C}_{1L}$ and $\mathfrak{C}_{2L}$ be two LVPF sets in $\mathcal{P}$. Then $\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L}$ and $\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L}$ are LVPF sets in $\mathcal{P}$.

Proof: The proof is clear.

Definition 3.8: The absolute LVPFS and null LVPFS are defined as $I_L = \{ \langle \dot{u}, 1_L, 0_L, 0_L \rangle \mid \dot{u} \in \mathcal{P} \}$ and $O_L = \{ \langle \dot{u}, 0_L, 0_L, 1_L \rangle \mid \dot{u} \in \mathcal{P} \}$ respectively.

Theorem 3.9: The absolute LVPFS and null LVPFS satisfy the following equalities:

$$1. I_L \cup O_L = I_L \quad (16)$$

$$2. I_L \cap O_L = O_L \quad (17)$$

$$3. \mathfrak{C}_L \cup I_L = I_L \quad (18)$$

$$4. \mathfrak{C}_L \cap O_L = O_L \quad (19)$$

$$5. \mathfrak{C}_L \cap I_L = \mathfrak{C}_L \cup O_L = (\zeta_{(\mathfrak{C}_L)}, 0_L, \vartheta_{(\mathfrak{C}_L)}) \quad (20)$$

Theorem 3.10: For $\mathfrak{C}_{1L}, \mathfrak{C}_{2L}, \mathfrak{C}_{3L} \in \text{LVPFS}(\mathcal{P})$, the following properties hold:

$$1. (\mathfrak{C}_{1L})^c = \mathfrak{C}_{1L} \text{ (Involution)}$$

$$2. \mathfrak{C}_{1L} \cup \mathfrak{C}_{1L} = \mathfrak{C}_{1L} \text{ and } \mathfrak{C}_{1L} \cap \mathfrak{C}_{1L} = \mathfrak{C}_{1L} \text{ ("Idempotency")}$$

$$3. \mathfrak{C}_{1L} \cup \mathfrak{C}_{2L} = \mathfrak{C}_{2L} \cup \mathfrak{C}_{1L} \text{ and } \mathfrak{C}_{1L} \cap \mathfrak{C}_{2L} = \mathfrak{C}_{2L} \cap \mathfrak{C}_{1L} \text{ (Commutativity)}$$

$$4. \mathfrak{C}_{1L} \cup (\mathfrak{C}_{2L} \cup \mathfrak{C}_{3L}) = (\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L}) \cup \mathfrak{C}_{3L} \text{ and}$$

$$5. \mathfrak{C}_{1L} \cap (\mathfrak{C}_{2L} \cap \mathfrak{C}_{3L}) = (\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L}) \cap \mathfrak{C}_{3L} \text{ (Associativity)}$$

$$6. \mathfrak{C}_{1L} \cup (\mathfrak{C}_{2L} \cap \mathfrak{C}_{3L}) = (\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L}) \cap (\mathfrak{C}_{1L} \cup \mathfrak{C}_{3L}) \text{ and}$$

$$7. \mathfrak{C}_{1L} \cap (\mathfrak{C}_{2L} \cup \mathfrak{C}_{3L}) = (\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L}) \cup (\mathfrak{C}_{1L} \cap \mathfrak{C}_{3L}) \text{ (Distributivity)}$$

$$8. (\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L})^c = \mathfrak{C}_{1L}^c \cap \mathfrak{C}_{2L}^c \text{ and } (\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L})^c = \mathfrak{C}_{1L}^c \cup \mathfrak{C}_{2L}^c \text{ (De Morgan's "Laws")}$$

Theorem 3.11: If $\mathfrak{C}_{1L}, \mathfrak{C}_{2L}, \mathfrak{C}_{3L} \in \text{LVPFS}(\mathcal{P})$ and $\mathfrak{C}_{1L} \subseteq \mathfrak{C}_{2L}$, then

$$1. \mathfrak{C}_{1L} \cap \mathfrak{C}_{3L} \subseteq \mathfrak{C}_{2L} \cap \mathfrak{C}_{3L} \quad (21)$$

$$2. \mathfrak{C}_{1L} \cup \mathfrak{C}_{3L} \subseteq \mathfrak{C}_{2L} \cup \mathfrak{C}_{3L} \quad (22)$$

Proof:

Since $\mathfrak{C}_{1L} \subseteq \mathfrak{C}_{2L}$, then, $\zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \leq \zeta_{(\mathfrak{C}_{2L})}(\dot{u}), \iota_{(\mathfrak{C}_{1L})}(\dot{u}) \leq \iota_{(\mathfrak{C}_{2L})}(\dot{u}), \vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) \geq \vartheta_{(\mathfrak{C}_{2L})}(\dot{u})$.

$$1. \zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \leq \zeta_{(\mathfrak{C}_{2L})}(\dot{u})$$

$$\Rightarrow \zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \zeta_{(\mathfrak{C}_{3L})}(\dot{u}) \leq \zeta_{(\mathfrak{C}_{2L})}(\dot{u}) \wedge \zeta_{(\mathfrak{C}_{3L})}(\dot{u})$$

$$\Rightarrow \zeta_{(\mathfrak{C}_{1L} \cap \mathfrak{C}_{3L})}(\dot{u}) \leq \zeta_{(\mathfrak{C}_{2L} \cap \mathfrak{C}_{3L})}(\dot{u})$$

$$\begin{aligned} \iota_{(\mathfrak{G}_{1L})}(\dot{u}) &\leq \iota_{(\mathfrak{G}_{2L})}(\dot{u}) \\ \Rightarrow \iota_{(\mathfrak{G}_{1L})}(\dot{u}) \wedge \iota_{(\mathfrak{G}_{3L})}(\dot{u}) &\leq \iota_{(\mathfrak{G}_{2L})}(\dot{u}) \wedge \iota_{(\mathfrak{G}_{3L})}(\dot{u}) \\ \Rightarrow \iota_{(\mathfrak{G}_{1L} \cap \mathfrak{G}_{3L})}(\dot{u}) &\leq \iota_{(\mathfrak{G}_{2L} \cap \mathfrak{G}_{3L})}(\dot{u}) \\ \vartheta_{(\mathfrak{G}_{1L})}(\dot{u}) &\geq \vartheta_{(\mathfrak{G}_{2L})}(\dot{u}) \\ \Rightarrow \vartheta_{(\mathfrak{G}_{1L})}(\dot{u}) \vee \vartheta_{(\mathfrak{G}_{3L})}(\dot{u}) &\geq \vartheta_{(\mathfrak{G}_{2L})}(\dot{u}) \vee \vartheta_{(\mathfrak{G}_{3L})}(\dot{u}) \\ \Rightarrow \vartheta_{(\mathfrak{G}_{1L} \cap \mathfrak{G}_{3L})}(\dot{u}) &\geq \vartheta_{(\mathfrak{G}_{2L} \cap \mathfrak{G}_{3L})}(\dot{u}) \end{aligned}$$

Thus, $\mathfrak{G}_{1L} \cap \mathfrak{G}_{3L} \subseteq \mathfrak{G}_{2L} \cap \mathfrak{G}_{3L}$.

2. The proof is comparable to (i).

Theorem 3.12: For any two LVPFSs $\mathfrak{G}_{1L}$ and $\mathfrak{G}_{2L}$ in $\wp$, $\mathfrak{G}_{1L} \cap \mathfrak{G}_{2L} \subseteq \mathfrak{G}_{1L}, \mathfrak{G}_{2L}$.

Proof:

Let $\dot{u} \in \mathcal{U}$. Then,

$$\begin{aligned} \zeta_{(\mathfrak{G}_{1L} \cap \mathfrak{G}_{2L})}(\dot{u}) &= \zeta_{(\mathfrak{G}_{1L})}(\dot{u}) \wedge \zeta_{(\mathfrak{G}_{2L})}(\dot{u}) \leq \zeta_{(\mathfrak{G}_{1L})}(\dot{u}), \zeta_{(\mathfrak{G}_{2L})}(\dot{u}) \\ \iota_{(\mathfrak{G}_{1L} \cap \mathfrak{G}_{2L})}(\dot{u}) &= \iota_{(\mathfrak{G}_{1L})}(\dot{u}) \wedge \iota_{(\mathfrak{G}_{2L})}(\dot{u}) \leq \iota_{(\mathfrak{G}_{1L})}(\dot{u}), \iota_{(\mathfrak{G}_{2L})}(\dot{u}) \\ \vartheta_{(\mathfrak{G}_{1L} \cap \mathfrak{G}_{2L})}(\dot{u}) &= \vartheta_{(\mathfrak{G}_{1L})}(\dot{u}) \vee \vartheta_{(\mathfrak{G}_{2L})}(\dot{u}) \geq \vartheta_{(\mathfrak{G}_{1L})}(\dot{u}), \vartheta_{(\mathfrak{G}_{2L})}(\dot{u}) \end{aligned}$$

Thus, $\mathfrak{G}_{1L} \cap \mathfrak{G}_{2L} \subseteq \mathfrak{G}_{1L}, \mathfrak{G}_{2L}$.

Theorem 3.13: $\mathfrak{G}_{1L} \cap \mathfrak{G}_{2L} = \mathfrak{G}_{1L}$ if and only if $\mathfrak{G}_{1L} \subseteq \mathfrak{G}_{2L}$.

Proof:

Let $\mathfrak{G}_{1L} \cap \mathfrak{G}_{2L} = \mathfrak{G}_{1L}$. Since $\mathfrak{G}_{1L} \cap \mathfrak{G}_{2L} \subseteq \mathfrak{G}_{2L}$, we have $\mathfrak{G}_{1L} \subseteq \mathfrak{G}_{2L}$.

Conversely, suppose that $\mathfrak{G}_{1L} \subseteq \mathfrak{G}_{2L}$.

Then, $\zeta_{(\mathfrak{G}_{1L})}(\dot{u}) \leq \zeta_{(\mathfrak{G}_{2L})}(\dot{u})$, $\iota_{(\mathfrak{G}_{1L})}(\dot{u}) \leq \iota_{(\mathfrak{G}_{2L})}(\dot{u})$, $\vartheta_{(\mathfrak{G}_{1L})}(\dot{u}) \geq \vartheta_{(\mathfrak{G}_{2L})}(\dot{u})$.

$$\begin{aligned} \zeta_{(\mathfrak{G}_{1L} \cap \mathfrak{G}_{2L})}(\dot{u}) &= \zeta_{(\mathfrak{G}_{1L})}(\dot{u}) \wedge \zeta_{(\mathfrak{G}_{2L})}(\dot{u}) = \zeta_{(\mathfrak{G}_{1L})}(\dot{u}) \\ \iota_{(\mathfrak{G}_{1L} \cap \mathfrak{G}_{2L})}(\dot{u}) &= \iota_{(\mathfrak{G}_{1L})}(\dot{u}) \wedge \iota_{(\mathfrak{G}_{2L})}(\dot{u}) = \iota_{(\mathfrak{G}_{1L})}(\dot{u}) \\ \vartheta_{(\mathfrak{G}_{1L} \cap \mathfrak{G}_{2L})}(\dot{u}) &= \vartheta_{(\mathfrak{G}_{1L})}(\dot{u}) \vee \vartheta_{(\mathfrak{G}_{2L})}(\dot{u}) = \vartheta_{(\mathfrak{G}_{1L})}(\dot{u}) \end{aligned}$$

Thus, $\mathfrak{G}_{1L} \cap \mathfrak{G}_{2L} = \mathfrak{G}_{1L}$.

Theorem 3.14: $\mathfrak{G}_{1L} \subseteq \mathfrak{G}_{1L} \cup \mathfrak{G}_{2L}$ iff $\iota_{(\mathfrak{G}_{1L})}(\dot{u}) \leq \iota_{(\mathfrak{G}_{2L})}(\dot{u}) \forall \dot{u} \in \wp$.

Proof:

Let $\mathfrak{G}_{1L} \subseteq \mathfrak{G}_{1L} \cup \mathfrak{G}_{2L}$.

Then,

$$\begin{aligned} \iota_{(\mathfrak{G}_{1L})}(\dot{u}) &\leq \iota_{(\mathfrak{G}_{1L} \cup \mathfrak{G}_{2L})}(\dot{u}) \\ &= \iota_{(\mathfrak{G}_{1L})}(\dot{u}) \wedge \iota_{(\mathfrak{G}_{2L})}(\dot{u}) \\ &\leq \iota_{(\mathfrak{G}_{2L})}(\dot{u}) \\ \Rightarrow \iota_{(\mathfrak{G}_{1L})}(\dot{u}) &\leq \iota_{(\mathfrak{G}_{2L})}(\dot{u}) \end{aligned}$$

Conversely, let $\iota_{(\mathfrak{G}_{1L})}(\dot{u}) \leq \iota_{(\mathfrak{G}_{2L})}(\dot{u})$. Then,

$$\begin{aligned} \zeta_{(\mathfrak{G}_{1L} \cup \mathfrak{G}_{2L})}(\dot{u}) &= \zeta_{(\mathfrak{G}_{1L})}(\dot{u}) \vee \zeta_{(\mathfrak{G}_{2L})}(\dot{u}) \geq \zeta_{(\mathfrak{G}_{1L})}(\dot{u}) \\ \iota_{(\mathfrak{G}_{1L} \cup \mathfrak{G}_{2L})}(\dot{u}) &= \iota_{(\mathfrak{G}_{1L})}(\dot{u}) \wedge \iota_{(\mathfrak{G}_{2L})}(\dot{u}) = \iota_{(\mathfrak{G}_{1L})}(\dot{u}) \\ \vartheta_{(\mathfrak{G}_{1L} \cup \mathfrak{G}_{2L})}(\dot{u}) &= \vartheta_{(\mathfrak{G}_{1L})}(\dot{u}) \wedge \vartheta_{(\mathfrak{G}_{2L})}(\dot{u}) \leq \vartheta_{(\mathfrak{G}_{1L})}(\dot{u}) \end{aligned}$$

Thus, $\mathfrak{G}_{1L} \subseteq \mathfrak{G}_{1L} \cup \mathfrak{G}_{2L}$.

Theorem 3.15: $\mathfrak{G}_{1L} \subseteq \mathfrak{G}_{1L} \cup \mathfrak{G}_{2L}$ and $\mathfrak{G}_{2L} \subseteq \mathfrak{G}_{1L} \cup \mathfrak{G}_{2L}$ iff $\iota_{(\mathfrak{G}_{1L})}(\dot{u}) = \iota_{(\mathfrak{G}_{2L})}(\dot{u}) \forall \dot{u} \in \wp$.

Proof: The proof is a consequence of Theorem 3.14.

Theorem 3.16: Let $\iota_{(\mathfrak{C}_{1L})}(\dot{u}) = \iota_{(\mathfrak{C}_{2L})}(\dot{u})$ for every $\dot{u} \in \wp$. Then, $\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L} = \mathfrak{C}_{2L}$ iff $\mathfrak{C}_{1L} \subseteq \mathfrak{C}_{2L}$.

Proof: Let $\iota_{(\mathfrak{C}_{1L})}(\dot{u}) = \iota_{(\mathfrak{C}_{2L})}(\dot{u})$.

$$\begin{aligned}\mathfrak{C}_{1L} \subseteq \mathfrak{C}_{2L} &\Leftrightarrow \zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \leq \zeta_{(\mathfrak{C}_{2L})}(\dot{u}), \iota_{(\mathfrak{C}_{1L})}(\dot{u}) \leq \iota_{(\mathfrak{C}_{2L})}(\dot{u}), \vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) \geq \vartheta_{(\mathfrak{C}_{2L})}(\dot{u}) \\ &\Leftrightarrow \zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \vee \zeta_{(\mathfrak{C}_{2L})}(\dot{u}) = \zeta_{(\mathfrak{C}_{2L})}(\dot{u}), \iota_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \iota_{(\mathfrak{C}_{2L})}(\dot{u}) = \iota_{(\mathfrak{C}_{1L})}(\dot{u}), \\ &\quad \vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \vartheta_{(\mathfrak{C}_{2L})}(\dot{u}) = \vartheta_{(\mathfrak{C}_{2L})}(\dot{u}) \\ &\Leftrightarrow \zeta_{(\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L})}(\dot{u}) = \zeta_{(\mathfrak{C}_{2L})}(\dot{u}), \iota_{(\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L})}(\dot{u}) = \iota_{(\mathfrak{C}_{1L})}(\dot{u}) = \iota_{(\mathfrak{C}_{2L})}(\dot{u}), \\ &\quad \vartheta_{(\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L})}(\dot{u}) = \vartheta_{(\mathfrak{C}_{2L})}(\dot{u}) \\ &\Leftrightarrow \mathfrak{C}_{1L} \cup \mathfrak{C}_{2L} = \mathfrak{C}_{2L}\end{aligned}$$

Theorem 3.17: Let $\mathfrak{C}_{1L}, \mathfrak{C}_{2L} \in LVPFS(\wp)$. Then, $\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L} = \mathfrak{C}_{1L} \cap \mathfrak{C}_{2L} = \mathfrak{C}_{1L}$ if $\mathfrak{C}_{1L} = \mathfrak{C}_{2L}$.

Proof: The proof is clear.

Remark 3.18: Theorem 3.17's converse is not true. Consider two LVPFSs $\mathfrak{C}_{1L}$ and $\mathfrak{C}_{2L}$ such that

$$\zeta_{(\mathfrak{C}_{1L})}(\dot{u}) = \zeta_{(\mathfrak{C}_{2L})}(\dot{u}), \iota_{(\mathfrak{C}_{1L})}(\dot{u}) \leq \iota_{(\mathfrak{C}_{2L})}(\dot{u}), \vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) = \vartheta_{(\mathfrak{C}_{2L})}(\dot{u}).$$

Clearly, $\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L} = \mathfrak{C}_{1L} \cap \mathfrak{C}_{2L} = \mathfrak{C}_{1L}$ but $\mathfrak{C}_{1L} \neq \mathfrak{C}_{2L}$.

Definition 3.19: A LVPFS $\mathfrak{C}_{1L}$ is equivalent to another LVPFS $\mathfrak{C}_{2L}$ (i.e) $\mathfrak{C}_{1L} \equiv \mathfrak{C}_{2L}$ if and only if

$$\zeta_{(\mathfrak{C}_{1L})}(\dot{u}) = \zeta_{(\mathfrak{C}_{2L})}(\dot{u}), \iota_{(\mathfrak{C}_{1L})}(\dot{u}) \leq \iota_{(\mathfrak{C}_{2L})}(\dot{u}), \vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) = \vartheta_{(\mathfrak{C}_{2L})}(\dot{u}). \quad (23)$$

Theorem 3.20: $\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L} = \mathfrak{C}_{1L} \cap \mathfrak{C}_{2L} = \mathfrak{C}_{1L}$ if and only if $\mathfrak{C}_{1L} \equiv \mathfrak{C}_{2L}$.

Proof: The proof is a consequence of Definition 3.19.

Theorem 3.21: Let $\mathfrak{C}_{1L}, \mathfrak{C}_{2L} \in LVPFS(\wp)$. Then,

$$1. \quad \mathfrak{C}_{1L} \cup (\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L}) \subseteq \mathfrak{C}_{1L} \quad (24)$$

$$2. \quad \mathfrak{C}_{1L} \cap (\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L}) \subseteq \mathfrak{C}_{1L} \quad (25)$$

Equality holds if $\iota_{(\mathfrak{C}_{1L})}(\dot{u}) \leq \iota_{(\mathfrak{C}_{2L})}(\dot{u})$.

Proof:

By Theorem 3.12, $\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L} \subseteq \mathfrak{C}_{1L}$.

$$\Rightarrow \zeta_{(\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L})}(\dot{u}) \leq \zeta_{(\mathfrak{C}_{1L})}(\dot{u}), \iota_{(\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L})}(\dot{u}) \leq \iota_{(\mathfrak{C}_{1L})}(\dot{u}), \vartheta_{(\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L})}(\dot{u}) \geq \vartheta_{(\mathfrak{C}_{1L})}(\dot{u})$$

$$\zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \vee \zeta_{(\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L})}(\dot{u}) = \zeta_{(\mathfrak{C}_{1L})}(\dot{u})$$

$$\iota_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \iota_{(\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L})}(\dot{u}) = \iota_{(\mathfrak{C}_{1L} \cup \mathfrak{C}_{2L})}(\dot{u}) \leq \iota_{(\mathfrak{C}_{1L})}(\dot{u})$$

$$\vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \vartheta_{(\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L})}(\dot{u}) = \vartheta_{(\mathfrak{C}_{1L})}(\dot{u})$$

$\Rightarrow$

$$\zeta_{(\mathfrak{C}_{1L} \cup (\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L}))}(\dot{u}) = \zeta_{(\mathfrak{C}_{1L})}(\dot{u}),$$

$$\iota_{(\mathfrak{C}_{1L} \cup (\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L}))}(\dot{u}) \leq \iota_{(\mathfrak{C}_{1L})}(\dot{u}),$$

$$\vartheta_{(\mathfrak{C}_{1L} \cup (\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L}))}(\dot{u}) = \vartheta_{(\mathfrak{C}_{1L})}(\dot{u})$$

Thus, $\mathfrak{C}_{1L} \cup (\mathfrak{C}_{1L} \cap \mathfrak{C}_{2L}) \subseteq \mathfrak{C}_{1L}$.

The proof is comparable to (i).

Definition 3.22: Let $\mathfrak{C}_{1L}, \mathfrak{C}_{2L} \in LVPFS(\wp)$. The difference between $\mathfrak{C}_{1L}$ and $\mathfrak{C}_{2L}$, denoted by $\mathfrak{C}_{1L} \setminus \mathfrak{C}_{2L}$ is defined as

$$\mathfrak{C}_{1L} \setminus \mathfrak{C}_{2L} = \{ \langle \dot{u}, \zeta_{(\mathfrak{C}_{1L} \setminus \mathfrak{C}_{2L})}(\dot{u}), \iota_{(\mathfrak{C}_{1L} \setminus \mathfrak{C}_{2L})}(\dot{u}), \vartheta_{(\mathfrak{C}_{1L} \setminus \mathfrak{C}_{2L})}(\dot{u}) \rangle \mid \dot{u} \in \wp \}$$

$$\zeta_{(\mathfrak{C}_{1L} \setminus \mathfrak{C}_{2L})}(\dot{u}) = \zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \vartheta_{(\mathfrak{C}_{2L})}(\dot{u}), \quad (26)$$

$$\iota_{(\mathfrak{C}_{1L} \setminus \mathfrak{C}_{2L})}(\dot{u}) = \iota_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \iota_{(\mathfrak{C}_{2L})}(\dot{u}), \quad (27)$$

$$\vartheta_{(\mathfrak{C}_{1L} \setminus \mathfrak{C}_{2L})}(\dot{u}) = \vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) \vee \zeta_{(\mathfrak{C}_{2L})}(\dot{u}). \quad (28)$$

Theorem 3.23: Let $\mathfrak{C}_{1L}, \mathfrak{C}_{2L} \in LVPFS(\wp)$. Then,

$$\text{i. } \mathfrak{C}_{1L} \setminus I_L = O_L \quad (29)$$

$$\text{ii. } \mathfrak{C}_{1L} \setminus \mathfrak{C}_{2L} = \mathfrak{C}_{1L} \cap \mathfrak{C}_{2L}^c \quad (30)$$

Proof:

$$\begin{aligned} 1. \quad \mathfrak{C}_{1L} \setminus I_L &= \{ \langle \dot{u}, \zeta_{(\mathfrak{C}_{1L})}(\dot{u}), \iota_{(\mathfrak{C}_{1L})}(\dot{u}), \vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) \rangle \mid \dot{u} \in \mathcal{U} \} \setminus \{ \langle \dot{u}, 1_L, 0_L, 0_L \rangle \mid \dot{u} \in \wp \} \\ &= \{ \langle \dot{u}, \zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge 0_L, \iota_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge 0_L, \vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) \vee 1_L \rangle \mid \dot{u} \in \wp \} \\ &= \{ \langle \dot{u}, 0_L, 0_L, 1_L \rangle \mid \dot{u} \in \wp \} \\ &= O_L \\ 2. \quad \mathfrak{C}_{1L} \cap \mathfrak{C}_{2L}^c &= \{ \langle \dot{u}, \zeta_{(\mathfrak{C}_{1L})}(\dot{u}), \iota_{(\mathfrak{C}_{1L})}(\dot{u}), \vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) \rangle \mid \dot{u} \in \wp \} \cap \\ &\quad \{ \langle \dot{u}, \vartheta_{(\mathfrak{C}_{2L})}(\dot{u}), \iota_{(\mathfrak{C}_{2L})}(\dot{u}), \zeta_{(\mathfrak{C}_{2L})}(\dot{u}) \rangle \mid \dot{u} \in \wp \} \\ &= \{ \langle \dot{u}, \zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \vartheta_{(\mathfrak{C}_{2L})}(\dot{u}), \iota_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \iota_{(\mathfrak{C}_{2L})}(\dot{u}), \vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) \vee \zeta_{(\mathfrak{C}_{2L})}(\dot{u}) \rangle \mid \dot{u} \in \wp \} \\ &= \mathfrak{C}_{1L} \setminus \mathfrak{C}_{2L} \end{aligned}$$

Definition 3.24: Let $\mathfrak{C}_{1L}, \mathfrak{C}_{2L} \in LVPFS(\wp)$. The symmetric difference between $\mathfrak{C}_{1L}$ and $\mathfrak{C}_{2L}$ is defined as

$$\mathfrak{C}_{1L} \Delta \mathfrak{C}_{2L} = \{ \langle \dot{u}, \zeta_{(\mathfrak{C}_{1L} \Delta \mathfrak{C}_{2L})}(\dot{u}), \iota_{(\mathfrak{C}_{1L} \Delta \mathfrak{C}_{2L})}(\dot{u}), \vartheta_{(\mathfrak{C}_{1L} \Delta \mathfrak{C}_{2L})}(\dot{u}) \rangle \mid \dot{u} \in \wp \} \text{ where} \quad (31)$$

$$\zeta_{(\mathfrak{C}_{1L} \Delta \mathfrak{C}_{2L})}(\dot{u}) = (\zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \vartheta_{(\mathfrak{C}_{2L})}(\dot{u})) \vee (\zeta_{(\mathfrak{C}_{2L})}(\dot{u}) \wedge \vartheta_{(\mathfrak{C}_{1L})}(\dot{u})), \quad (32)$$

$$\iota_{(\mathfrak{C}_{1L} \Delta \mathfrak{C}_{2L})}(\dot{u}) = \iota_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \iota_{(\mathfrak{C}_{2L})}(\dot{u}), \quad (33)$$

$$\vartheta_{(\mathfrak{C}_{1L} \Delta \mathfrak{C}_{2L})}(\dot{u}) = (\vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) \vee \zeta_{(\mathfrak{C}_{2L})}(\dot{u})) \wedge (\vartheta_{(\mathfrak{C}_{2L})}(\dot{u}) \vee \zeta_{(\mathfrak{C}_{1L})}(\dot{u})). \quad (33)$$

Theorem 3.25: Let $\mathfrak{C}_{1L}, \mathfrak{C}_{2L} \in LVPFS(\wp)$. Then,

$$1. \quad \mathfrak{C}_{1L} \Delta \mathfrak{C}_{2L} = \mathfrak{C}_{2L} \Delta \mathfrak{C}_{1L} \quad (34)$$

$$2. \quad \mathfrak{C}_{1L} \Delta \mathfrak{C}_{2L} = (\mathfrak{C}_{1L} \setminus \mathfrak{C}_{2L}) \cup (\mathfrak{C}_{2L} \setminus \mathfrak{C}_{1L}) \quad (35)$$

Proof:

1. The proof is clear.

$$\begin{aligned} 2. \quad (\mathfrak{C}_{1L} \setminus \mathfrak{C}_{2L}) \cup (\mathfrak{C}_{2L} \setminus \mathfrak{C}_{1L}) &= \{ \langle \dot{u}, \zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \vartheta_{(\mathfrak{C}_{2L})}(\dot{u}), \iota_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \iota_{(\mathfrak{C}_{2L})}(\dot{u}), \vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) \vee \zeta_{(\mathfrak{C}_{2L})}(\dot{u}) \rangle \mid \dot{u} \in \wp \} \cup \\ &\quad \{ \langle \dot{u}, \zeta_{(\mathfrak{C}_{2L})}(\dot{u}) \wedge \vartheta_{(\mathfrak{C}_{1L})}(\dot{u}), \iota_{(\mathfrak{C}_{2L})}(\dot{u}) \wedge \iota_{(\mathfrak{C}_{1L})}(\dot{u}), \vartheta_{(\mathfrak{C}_{2L})}(\dot{u}) \vee \zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \rangle \mid \dot{u} \in \wp \} \\ &= \{ \langle \dot{u}, (\zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \vartheta_{(\mathfrak{C}_{2L})}(\dot{u})) \vee (\zeta_{(\mathfrak{C}_{2L})}(\dot{u}) \wedge \vartheta_{(\mathfrak{C}_{1L})}(\dot{u})), \\ &\quad (\iota_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \iota_{(\mathfrak{C}_{2L})}(\dot{u})) \wedge (\iota_{(\mathfrak{C}_{2L})}(\dot{u}) \wedge \iota_{(\mathfrak{C}_{1L})}(\dot{u})), \\ &\quad (\vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) \vee \zeta_{(\mathfrak{C}_{2L})}(\dot{u})) \wedge (\vartheta_{(\mathfrak{C}_{2L})}(\dot{u}) \vee \zeta_{(\mathfrak{C}_{1L})}(\dot{u})) \rangle \mid \dot{u} \in \wp \} \\ &= \{ \langle \dot{u}, (\zeta_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \vartheta_{(\mathfrak{C}_{2L})}(\dot{u})) \vee (\zeta_{(\mathfrak{C}_{2L})}(\dot{u}) \wedge \vartheta_{(\mathfrak{C}_{1L})}(\dot{u})), (\iota_{(\mathfrak{C}_{1L})}(\dot{u}) \wedge \iota_{(\mathfrak{C}_{2L})}(\dot{u})), \\ &\quad (\vartheta_{(\mathfrak{C}_{1L})}(\dot{u}) \vee \zeta_{(\mathfrak{C}_{2L})}(\dot{u})) \wedge (\vartheta_{(\mathfrak{C}_{2L})}(\dot{u}) \vee \zeta_{(\mathfrak{C}_{1L})}(\dot{u})) \rangle \mid \dot{u} \in \wp \} \\ &= \mathfrak{C}_{1L} \Delta \mathfrak{C}_{2L} \end{aligned}$$

IV. CONCLUSION

The LVPFS has been described and the properties of set operations are studied in this article. The work can be extended to address issues with decision making.

REFERENCES

1. L. A. Zadeh, "Fuzzy Sets", Information and Control (Science China Press, China, 1965), 8(3), pp. 338-353. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)
2. K. T. Atanassov, "Intuitionistic fuzzy sets", Fuzzy Sets and Systems (Elsevier B.V, Netherlands, 1986), 20(1), pp. 87-96. [https://doi.org/10.1016/S0165-0114\(86\)80034-3](https://doi.org/10.1016/S0165-0114(86)80034-3)
3. F. Smarandache, "Neutrosophy: Neutrosophic Probability, Set, and Logic: Analytic Synthesis & Synthetic Analysis" (American Research Press, Rehoboth, 1998). <https://books.google.co.in/books?id=LgEZAQAIAAJ>

4. B. C. Cuong, "Picture Fuzzy Sets", Journal of Computer Science and Cybernetics (Naim Cagman, Turkey, 2014), 30(4), pp. 409-420. <https://doi.org/10.15625/1813-9663/30/4/5032>
5. Florentin Smarandache, "Neutrosophic Set is a Generalization of Intuitionistic Fuzzy Set, Inconsistent Intuitionistic Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set (Atanassov's Intuitionistic Fuzzy Set of second type), q-Rung Orthopair Fuzzy Set, Spherical Fuzzy Set, and n-HyperSpherical Fuzzy Set, while Neutrosophication is a Generalization of Regret Theory, Grey System Theory, and Three-Ways Decision (revisited)", Journal of New Theory (Walter de Gruyter, Berlin, 2019), 29, pp. 01-35. <https://doi.org/10.48550/arXiv.1911.07333>
6. Yuan Bo, Wu Wangming, "Fuzzy ideals on a distributive lattice", Fuzzy Sets and Systems (Elsevier B.V, Netherlands, 1990), 35(2), pp. 231-240. [https://doi.org/10.1016/0165-0114\(90\)90196-D](https://doi.org/10.1016/0165-0114(90)90196-D)
7. T. Gerstenkorn, A. Tepavčević, "Lattice valued intuitionistic fuzzy sets", Central European Journal of Mathematics (2004), 2(3), pp. 388-398. <https://doi.org/10.2478/BF02475236>
8. Jose James, Sunil C. Mathew, "Lattice valued neutrosophic sets", Journal of Mathematical and Computational Science (SCI Publishing Corporation, UK, 2021), 11(4), pp. 4695 – 4710.
9. Subrata, Bhowmik, "Lattice and Topology" (2014). <http://dx.doi.org/10.13140/RG.2.1.1384.6005>
10. Y. M. Liu, M. K. Luo, "Fuzzy topology", Advances in Fuzzy Systems - Applications And Theory (World Scientific Publishing Co Pte Ltd, Singapore, 1997). <https://books.google.co.in/books?id=CZjsCgAAQBAJ>



INTERNATIONAL
STANDARD
SERIAL
NUMBER
INDIA



INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

 9940 572 462  6381 907 438  ijirset@gmail.com



www.ijirset.com

Scan to save the contact details